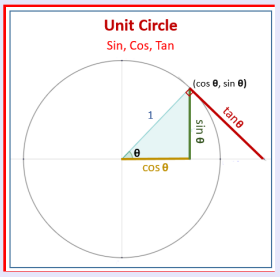
Trigonometry

Lecture 37



Feb 19-8:47 AM

More of trig. equations:

Solve $4\sin x - 3 = 0$ over $[0°, 360°)$

$\sin x = \frac{3}{4}$, QI, QII

R.A. $\sin^{-1}\left(\frac{3}{4}\right) \approx 49°$

QI: $x = RA$ $x = 49°$

QII: $x = 180° - RA$ $x = 131°$

Solve $2\tan 2x + 1 = 0$ over $[0°, 360°)$

$\tan 2x = -\frac{1}{2}$ QII & QIV

Ref. Angle $\tan^{-1}\left(\frac{1}{2}\right) \approx 27°$

QII $2x = 180° - 27° + K \cdot 180°$

$\boxed{x = 76.5° + K \cdot 90°}$ K=0 → 76.5°, 166.5°

QIV $2x = 360° - 27° + K \cdot 180°$

$\boxed{x = 166.5° + K \cdot 90°}$ K=1 → ~~166.5°~~, 256.5°

K=2 → ~~256.5°~~, 346.5°

$\{76.5°, 166.5°, 256.5°, 346.5°\}$

Nov 5-10:28 AM

Solve $1 + \sin\theta = 2\cos^2\theta$ $[0, 2\pi)$

Recall
$\sin^2\theta + \cos^2\theta = 1 \rightarrow \cos^2\theta = 1 - \sin^2\theta$

$1 + \sin\theta = 2(1 - \sin^2\theta)$

$1 + \sin\theta = 2 - 2\sin^2\theta$

$1 + \sin\theta - 2 + 2\sin^2\theta = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

By Zero-Product Rule

$2\sin\theta - 1 = 0$ $\qquad$ $\sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ $\qquad$ $\sin\theta = -1$

QI, QII $\qquad$ Quadrantal Angle

RA $\frac{\pi}{6}$ $\qquad$ $\theta = \frac{3\pi}{2}$

QI $\theta = \frac{\pi}{6}$, QII $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$

General Solutions

$\theta = \frac{\pi}{6} + k \cdot 2\pi$

$\theta = \frac{5\pi}{6} + k \cdot 2\pi$

$\theta = \frac{3\pi}{2} + k \cdot 2\pi$

Nov 5-10:38 AM

Solve $1 + \cos\theta = \sin\theta$ in $[0, 2\pi)$.

Square both sides

$(1 + \cos\theta)^2 = \sin^2\theta$

$(1 + \cos\theta)(1 + \cos\theta) = \sin^2\theta$

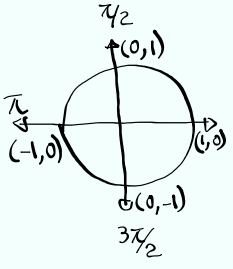
$1 + \cos\theta + \cos\theta + \cos^2\theta = 1 - \cos^2\theta$

$\cos^2\theta + 2\cos\theta + \cos^2\theta = 0$

$2\cos^2\theta + 2\cos\theta = 0$

$\cos^2\theta + \cos\theta = 0$

$\cos\theta(\cos\theta + 1) = 0$



$\cos\theta = 0$ $\qquad$ $\cos\theta = -1$

$\frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ π

$\left\{ \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\}$

Nov 5-10:46 AM

Solve $2 \sin 3x - 1 = 0$

Give general solution.

$\sin 3x = \frac{1}{2}$ QI, QII

RA. 30°

QI $3x = 30^\circ + K \cdot 360^\circ \rightarrow \boxed{x = 10^\circ + K \cdot 120^\circ}$

QII $3x = 180^\circ - 30^\circ + K \cdot 360^\circ \rightarrow \boxed{x = 50^\circ + K \cdot 120^\circ}$

Nov 5-10:53 AM

Solve $\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = 1$

$\sin 30^\circ$ $\cos 30^\circ$

$\sin 30^\circ \cos x + \cos 30^\circ \sin x = 1$

$\sin(30^\circ + x) = 1$

Recall $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(30^\circ + x) = 1$

RA. 90°

$30^\circ + x = 90^\circ + K \cdot 360^\circ$

$\boxed{x = 60^\circ + K \cdot 360^\circ}$

Nov 5-10:57 AM

Solve $\sin 3x + \cos 3x = \sqrt{2}$ over $[0°, 360°)$

Divide by $\sqrt{2}$

$$\frac{1}{\sqrt{2}}\sin 3x + \frac{1}{\sqrt{2}}\cos 3x = 1$$

$\frac{1}{\sqrt{2}} = \cos 45°$, $\frac{1}{\sqrt{2}} = \sin 45°$

$$\cos 45° \sin 3x + \sin 45° \cos 3x = 1$$

$$\sin(3x + 45°) = 1$$

$$3x + 45° = 90° + k \cdot 360°$$

$$3x = 45° + k \cdot 360°$$

$$x = 15° + k \cdot 120°$$

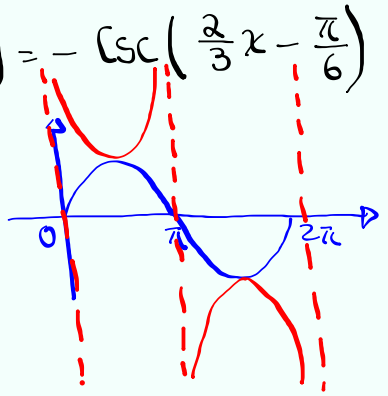
$k=0$	$x=15°$	$k=3$ Not in $[0°, 360°)$
$k=1$	$x=135°$	
$k=2$	$x=255°$	$\{15°, 135°, 255°\}$

Nov 5-11:02 AM

Graph $y = -\csc\left(\frac{2}{3}x - \frac{\pi}{6}\right)$

$y = \sin x$

$y = \csc x$

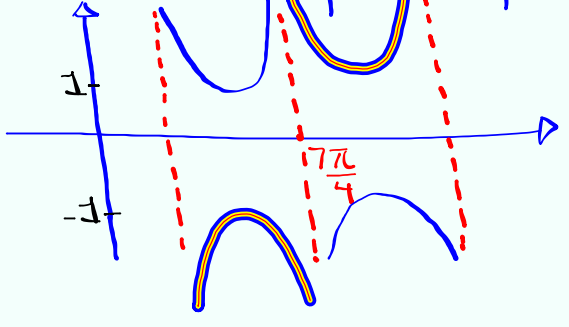


$0 < \frac{2}{3}x - \frac{\pi}{6} < 2\pi$

$0 < 4x - \pi < 12\pi$

$\pi < 4x < 13\pi$

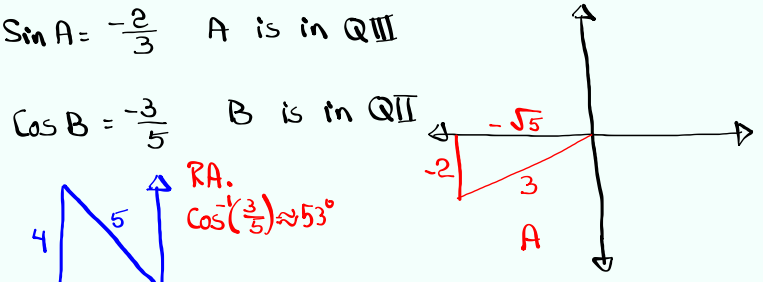
$\frac{\pi}{4} < x < \frac{13\pi}{4}$



Nov 5-11:09 AM

$\sin A = -\frac{2}{3}$ A is in QIII

$\cos B = -\frac{3}{5}$ B is in QII



$B = 180° - 53° = 127°$

Find $\cos(A - B)$

$\cos A \cos B + \sin A \sin B$

$-\frac{\sqrt{5}}{3} \cdot \frac{-3}{5} + \frac{-2}{3} \cdot \frac{4}{5}$

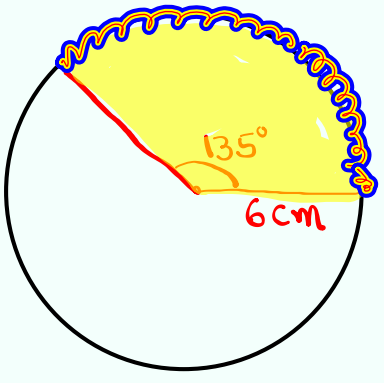
$= \frac{3\sqrt{5} - 8}{15}$

$\tan \frac{B}{2} = \frac{1 - \cos B}{\sin B} = \frac{1 - \frac{-3}{5}}{\frac{4}{5}}$

$\tan\left(\frac{127}{2}\right) \approx 2.00567\ldots = \frac{5+3}{4} = \frac{8}{4} = 2$

Nov 5-11:14 AM

Consider a Central angle of 135° with radius of 6 cm.



$135° = 180° - 45°$

$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ Rad.

Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \cdot 6^2 \cdot \frac{3\pi}{4}$

$= 13.5\pi \text{ cm}^2$

$= \frac{27\pi}{2} \text{ cm}^2$

$S = r\theta = 6 \cdot \frac{3\pi}{4}$

$= \frac{9\pi}{2}$ cm

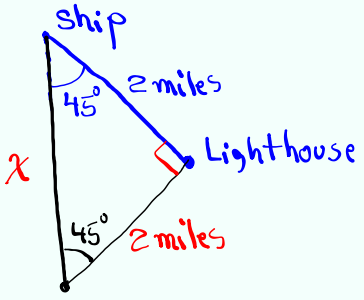
Nov 5-11:22 AM

A ship is 2 miles from a lighthouse with a bearing of S 45° E.

Ship is sailing South,

The bearing now is N 45° E.

How far the ship has sailed South?



$x^2 = 2^2 + 2^2$

$x^2 = 8$

$x = \sqrt{8}$

$\boxed{x \approx 3}$ miles.

Nov 5-11:27 AM

The tip of a blade of a fan rotates 20 times in 45 seconds. It is 1.5 ft long.

Find angular velocity in Rad/min.

$$w = \frac{\theta}{t} = \frac{20 \cdot 2\pi}{45 \text{ Seconds}} \cdot \frac{60 \text{ Seconds}}{1 \text{ Min.}}$$

$$= \frac{160\pi}{3} \text{ Rad/min.}$$

Find linear speed ft/hr.

$$v = r w = 1.5 \text{ ft} \cdot \frac{160\pi}{3} \frac{\text{Rad}}{\text{Min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}}$$

$$= 4800\pi \text{ ft/hr} \approx 15080 \text{ ft/hr}$$

Nov 5-11:32 AM